University of California, Berkeley
Physics H7C Fall 2002 (*Strovink*)

PROBLEM SET 8

1.
Prove that the presence of an *imaginary* potential causes the probability density $\rho = \psi^*\psi$ *not* to be conserved. That is, show that ρ *fails* to satisfy a continuity equation involving the probability current

$$j \equiv \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right).$$

2.
A particle is confined to the region $-L/2 < x < L/2$ by an infinitely deep potential well.
(a.)
What is the probability that the particle is found in the region $-L/2 < x < 0$? Does this probability depend on the quantum number n?
(b.)
If the particle is in the ground state, compute the probability that it is found in the central half of the box, $-L/4 < x < L/4$. How does this probability change if the particle is in a much higher energy state?

3.
A particle of mass m is bound in an infinitely deep one-dimensional potential well extending from $x = 0$ to $x = L$. At $t = 0$ it is described by a wavefunction of the form

$$u(x) \propto \sin\left(\pi x/L\right) + 2\sin\left(2\pi x/L\right).$$

(a.)
Normalize $u(x)$.
(b.)
What is the expectation value $\langle E \rangle$ of the particle's kinetic energy? (Do an integral to obtain this result.)
(c.)
When the particle's kinetic energy E is measured *for the first time*, what values could be obtained, and with what probability? Is your answer consistent with the result of (b.)?

(d.)
After this first measurement, $\langle E \rangle$ is redetermined. What value(s) could be obtained?

4.
This is a continuation of problem (**3**) [ignore **3**(c.) and **3**(d.)].
(a.)
At $t = 0$, calculate the expectation value $\langle x \rangle$ of the particle's position in the well (use brute force integration).
(b.)
Given $u(x) \equiv \psi(x,0)$ from problem **3**(a.), write down the time-dependent wavefunction $\psi(x,t)$.
(c.)
Your result for (a.) is the *minimum* value that $\langle x \rangle$ can take (why?). What is the earliest time at which $\langle x \rangle$ will reach a *maximum* value? (Cogent arguments can substitute for brute force algebra here, and are encouraged.)

5. *Estimating the strength of the strong force.*
A proton is confined to a nucleus that has a radius of 2 fm. (Work in one dimension.)
(a.)
Use the uncertainty principle (Bernstein Eqs. (7-25,26,27)) to estimate the proton's kinetic energy.
(b.)
Consider the proton to behave like a classical harmonic oscillator (force proportional to displacement) with a maximum displacement of 2 fm. Calculate the strength of the force on the proton (in MeV/fm) at its maximum displacement.
(c.)
In the same units, calculate the electric force between two protons separated by 2 fm, and compare it with your answer to (b.).

6.
Bernstein 6-23.

7.
Bernstein 7-12.

8.
Bernstein 7-21.